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A heuristic for nesting problems of irregular shapes

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Abstract

Layout has a close relationship with product cost in the vein of how to most efficiently cut product patterns from raw materials. This is the so-called "nesting problem", which occurs frequently in sheet metal and furniture industries, wherein material utilization needs to be maximized. In this paper, a quick location and movement (QLM) algorithm is proposed to solve the situation of irregular shapes nested on multiple irregular sheets. This approach includes two major parts: it first approximates irregular shapes to a polygon with the use of a cluster of straight lines, and second, it arranges the approximated shapes one-by-one with the proposed step-by-step rule. Finally, this study investigates and compares examples presented by other authors. The results show that the QLM algorithm takes less time to calculate a layout and the material utilization efficiency is higher compared to other methods.

Keywords: Nesting; Stock cutting; Material utilization; Pattern layout

1. Introduction

Nesting is a classic problem of finding the most efficient layout for cutting parts out of a given sheet with minimum waste material. The number of applications, e.g. steel, clothing, shipbuilding, and furniture industry is numerous. Manual methods are used to determine the arrangement in some industries. Operators decide the layout from their experience, but this is not an efficient method because it is time-consuming and the results do not efficiently utilize the raw material. It is challenging to obtain an efficient solution in a reasonable time when there are a large number of parts.

Nesting is an NP-hard problem and an optimal solution is impossible to calculate in a timely manner. Hence, a result is considered acceptable in practice if it is adequate and can be quickly obtained. Many researchers have attempted to develop methods or algorithms for nesting different shaped parts on different shaped sheets.

Several researchers have attempted to develop methods for nesting rectangular-shaped parts on rectangular sheets. Gimor and Gomory [7] use mathematical techniques to solve this kind of problem. Grinde and Cavalier [10] also used mathematical programming to contain a single polygon for solving nesting problems. Unfortunately, mathematical programming is not suitable in the case of many kinds of shapes. A class of heuristics introduced by Israni and Sanders [12] has been applied to the same problem. A genetic algorithm for placing polygons on a rectangular board is proposed by Jakobs [13].

Adamowicz and Albano [1] and Ismail and Sanders [11] group and cluster parts of irregular shapes to approximate rectangular shapes. They then arrange the grouped shapes by using rectangular nesting methods. These methodologies lower the degree of difficulty associated with these problems, but they also reduce the utilization of material because the approximated rectangular shapes are larger than the original irregular ones.

Albano and Sapuppo [3] attempt to address the nesting problem to a search-space process using heuristic methods. Gomes and Oliveira [8] also propose a heuristic approach that guides the search through the solution space. These methodologies require significant time to obtain an efficient solution. Many meta-heuristic methods have been attempted to solve nesting problems, such as simulated annealing by Lutfiyya et al. [15], tabu search by Bennell and Dowsland [5], and genetic algorithms by Anand et al. [2]. Even though these approaches can find efficient layouts, they are also similarly time-consuming like manual solutions by human operators.

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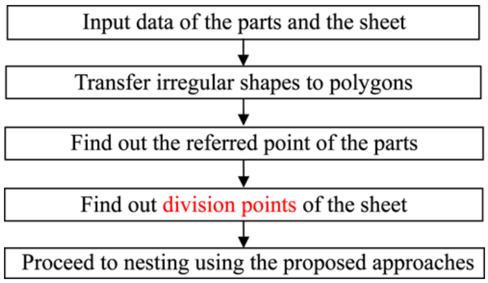

Fig. 1. The flow of the quick location and movement approach.

Babu and Babu [17] propose an approach employing both genetic and heuristic algorithms that aim to arrange different rectangular parts on multiple rectangular sheets. Wu et al. [20] also use a hybrid algorithm to solve cutting problems for two-dimensional rectangular parts on multiple plates. Furthermore, Lamousin et al. [14] develop the concept of no fit polygon (NFP) for the nesting of irregular parts on irregular sheets. An alternative approach for nesting irregular shapes proposed by Yousef [21] is based on extracting human intuitive thoughts. It is demonstrated that this approach yields efficient utilization performance that compares well with results achieved by expert human operators.

Grinde and Cavalier [9] and Nye [16] attempt to find an optimal solution for the nesting of convex shapes. Other researchers have also developed methods for polygon packing, e.g. Bennell et al. [4], Dowsland et al. [6], and Tay et al. [19]. In reality, shapes may contain straight lines and curvilinear features. Moreover, the parts may also contain internal features that are irregular in geometry. Apart from research by Babu and Babu [18], the above studies do not consider the situation where parts and sheets are multiple complexes with internal features or defective regions. Therefore, this research proposes the heuristic algorithm, quick location and movement (QLM), to solve nesting problems for multiple two-dimensional complex irregular parts on multiple two-dimensional complex irregular sheets.

2. Methodology

A quick location and movement (QLM) approach is presented by this work to solve the situation of nesting irregular shapes on multiple irregular sheets. The whole nesting flow is shown in Fig. 1, and each segment is introduced below.

2.1. Transformation of irregular shapes to polygon

It is challenging to calculate the movement and overlap of irregular shapes. Therefore, this research proposed a method to transform irregular shapes to polygons. The transformed polygon is slightly larger than the original irregular shape. An irregular shape includes straight lines and curve lines. Curved segments are presented as a set of arcs and the arcs are divided into convex and concave shapes. Taking Fig. 2 as an example, the curve ACB is composed of arc AC and arc BC.

$O_1(x_{01}, y_{01})$: the coordinates of the circle center of arc BC
$O_2(x_{02}, y_{02})$: the coordinates of the circle center of arc AC

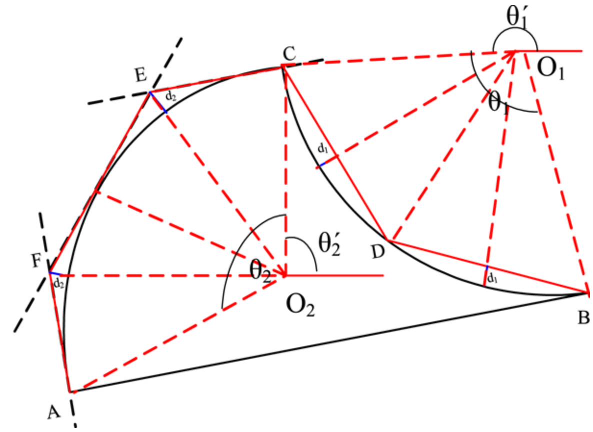

Fig. 2. An example of transforming irregular shape to polygon.

θ_1: the angle from line CO_1 to line BO_1
θ_1': the angle from line CO_1 to the horizontal line passing through O_1
θ_2: the angle from line CO_2 to line AO_2
θ_2': the angle from line CO_2 to the horizontal line passing through O_2
d_1: the longest distance from arc BD to line BD
d_2: the shortest distance from point E to arc AC
R_1: the radius of the circle center of arc BC
R_2: the radius of the circle center of arc AC
$A(x_A, y_A)$: the coordinates of point A

Coordinates of other points:

$$B(x_B, y_B) = (x_{01} + R_1 \cos(\theta_1 + \theta_1'), y_{01} + R_1 \sin(\theta_1 + \theta_1'))$$

$$\begin{aligned} D(x_D, y_D) &= \left(x_{01} + R_1 \cos\left(\frac{1}{2}\theta_1 + \theta_1'\right),\right. \\ &\quad \left. y_{01} + R_1 \sin\left(\frac{1}{2}\theta_1 + \theta_1'\right)\right) \end{aligned}$$

$$\begin{aligned} C(x_C, y_C) &= (x_{01} + R_1 \cos\theta_1', y_{01} + R_1 \sin\theta_1') \\ &= (x_{02} + R_2 \cos\theta_2', y_{02} + R_2 \sin\theta_2') \end{aligned}$$

$$\begin{aligned} E(x_E, y_E) &= \left(x_{02} + R_2 \csc\frac{1}{4}\theta_2 \left|\cos\left(\frac{1}{4}\theta_2 + \theta_{2'}'\right)\right|,\right. \\ &\quad \left. y_{02} + R_2 \csc\frac{1}{4}\theta_2 \sin\left(\frac{1}{4}\theta_2 + \theta_2'\right)\right) \end{aligned}$$

$$\begin{aligned} F(x_F, y_F) &= \left(x_{02} + R_2 \csc\frac{3}{4}\theta_2 \left|\cos\left(\frac{3}{4}\theta_2 + \theta_2'\right)\right|,\right. \\ &\quad \left. y_{02} + R_2 \csc\frac{3}{4}\theta_2 \sin\left(\frac{3}{4}\theta_2 + \theta_2'\right)\right). \end{aligned}$$

If the arc is cut by n segments, every coordinate of each concave segment is:

$$\begin{aligned} K(x_K, y_K) &= x_{0K} + R_K \cos\left(\pi - \left(\frac{m}{n}\theta_1 + \theta_1'\right)\right), \\ &\quad y_{0K} + R_K \sin\left(\pi - \left(\frac{m}{n}\theta_1 + \theta_1'\right)\right) \\ &\quad m = n-1, n-2, n-3, \ldots, 1. \end{aligned} \tag{1}$$

Every coordinate of each convex segment is:

$$J(x_J, y_J) = \left(x_{0J} + R_J \csc\left(\frac{2m-1}{2n}\theta_2\right)\right.$$

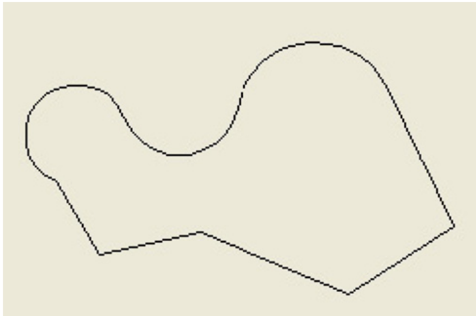

Fig. 3(a). Original shape.

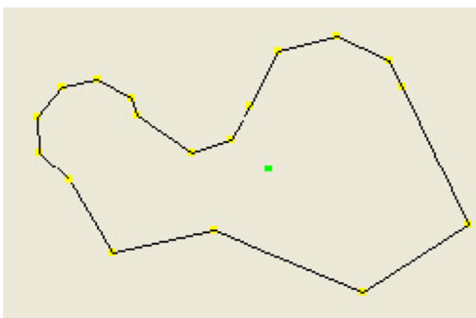

Fig. 3(b). Discrete with $\theta = 40°$.

Fig. 3(c). Discrete with $\theta = 20°$.

$$\left|\cos\left(\frac{2m-1}{2n}\theta_2+\theta_2'\right)\right|, y_{0J}+R_J\csc\left(\frac{2m-1}{2n}\theta_2\right) \times \sin\left(\frac{2m-1}{2n}\theta_2+\theta_2'\right)\Bigg) \quad m=1,2,3,\ldots,n. \tag{2}$$

For convenience of expression and calculation, the method uses a fixed θ, the angle of divided segment, as the parameter to approximate each original arc. The advantage of using a fixed angle is that only one parameter needs to be calculated for each arc, regardless of sheet size. Using the above equations, an original irregular shape can be transformed to a polygon; the more the segments, the more closely it resembles the original, as shown in Figs. 3(a)–3(c).

Although these formulas consider shapes with line and arc features, the discrete representation scheme adopted in this method can quickly and easily incorporate arbitrary features for parts and sheets.

2.2. Referral point

For the convenience of nesting, it is useful to determine the center of the least embedding circle and the center of gravity. An example is shown in Fig. 4. The center of the least embedding circle is beneficial for expressing the position and coordinates of parts. The center of gravity is used to decide the final position of parts.

2.3. Division points of sheets

For the purpose of convenience and quick location, it is useful to set the division points of sheets. As an example, a rectangular sheet is displayed in Fig. 5, wherein the left most bottom point is division point 1 and is assigned the coordinates (0, 0). Every division point is located at a distance of D. The point higher than the first one is division point 2. If no points are higher than the last point of the row, the point next to the right row is the next point and so on. The highest point in row 1 is division point 7, and division point 8 is the bottom point in row 2. Then, the algorithm records the coordinates and the sequence of every division points. In addition, a string of codes are recorded to the cover conditions of the division points. If a division point is covered with parts, it would be coded as a “1”, otherwise, as “0”.

2.4. Flow of arrangement

Fig. 6 shows the flow of the nest rules. At first, all parts are sorted by their area, and the largest is adopted as the first one. They are arranged piece by piece from the largest to the smallest, and the coordinates are recorded upon deciding the final position of each part. The parts are arranged with the nest rule introduced in the next section. The algorithm stops if it cannot find a position to arrange a part. The user must decide to choose another bigger sheet or to remove some parts, and then try again.

2.5. Nest rules

In the nest rules, the methodology represented by this study has two major parts. The first is how to search for a good initial position, and the second is how to move and rotate the shape and find the final position.

Some researchers put parts at the top-right corner to start, but this costs time to reach a good final position. This paper proposes a method to ascertain a good starting position without wasting long-term movement. In the first arranged part section, the position of division point, $T_1(p, p)$, is in its initial position if p is closest to the radius of the embedding circle of the part. Let the center of the embedding circle of the part move to the coordinates, $T_1(p, p)$. It then follows the flow and rule described in Fig. 7 to move and rotate. It records and compares every position moved and it rewrites the record if the location is better than the previous one. The judgment of location is dependent on the position of the center of gravity of the part. It is better if the position is closer to the origin of the coordinates, (0, 0).

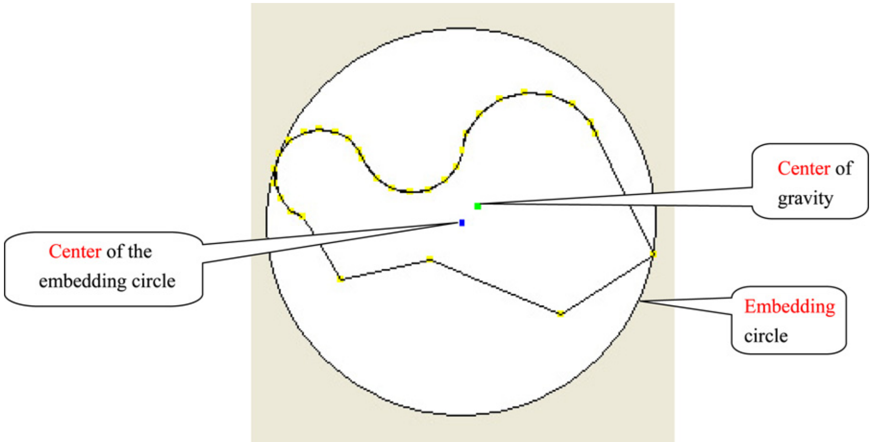

Fig. 4. Referral points of a part.

Fig. 5. Division points of a sheet.

The above-mentioned rule is designed for the initial position of the first part. The initial point selection of other parts is different from the first one. It chooses the division point where there are numbers of $(W - 1)$ of the division point without overlapping in four directions as the initial position of the next part. The value of W is calculated using Eq. (3). All parts move, rotate, and decide the final position in the same way. Every division point will be attempted without overlap if none fits the above condition.

$$\frac{L}{D} = W + \frac{t}{D}, \quad (W, t \in Z^{+}, t < D) \tag{3}$$
$$D = \alpha L$$
$$d = \beta D$$

L: the diameter of the surrounding circle of the part

D: the distance of every division point

α: coefficient

β: coefficient.

2.6. Multiple irregular sheets with defective regions

The preceding section aimed to arrange irregular parts on a rectangular sheet. In the case of an irregular sheet, it needs to be processed before applying the aforementioned methods. The aim here is to find the least embedding rectangle for the irregular sheet. Using the methods outlined in Section 2.1, areas of the rectangle outside the irregular shape are treated as shapes that have been already fitted. Similarly, any defects in the sheet are also considered as shapes that have been already placed. The parts then can be nested with the previously proposed approaches above on the unoccupied regions.

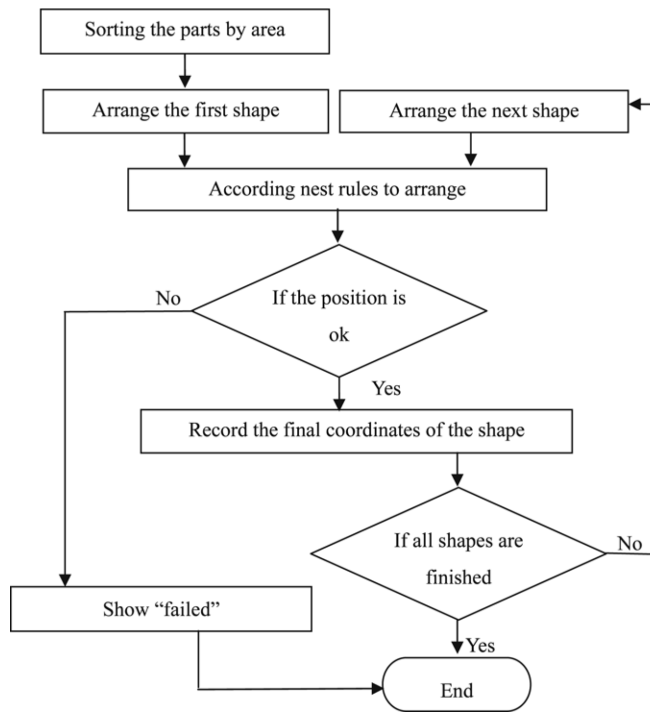

Fig. 6. The flow chart of arrangement.

In the case of multiple sheets, the sheets are sorted by area and combined in a raw sequence to find out the least embedding rectangle. Similarly, the extra and defective regions are recorded and viewed as occupied parts. This method can solve the problem of multiple irregular parts and sheets with or without defective areas.

3. Test results

Trials were initially carried out to test the influences of the parameters, θ, α, and β. These were executed by nesting forty pieces of a single part on a rectangular sheet with a fixed width and 'infinite' length, such as a bolt of cloth. All test runs were

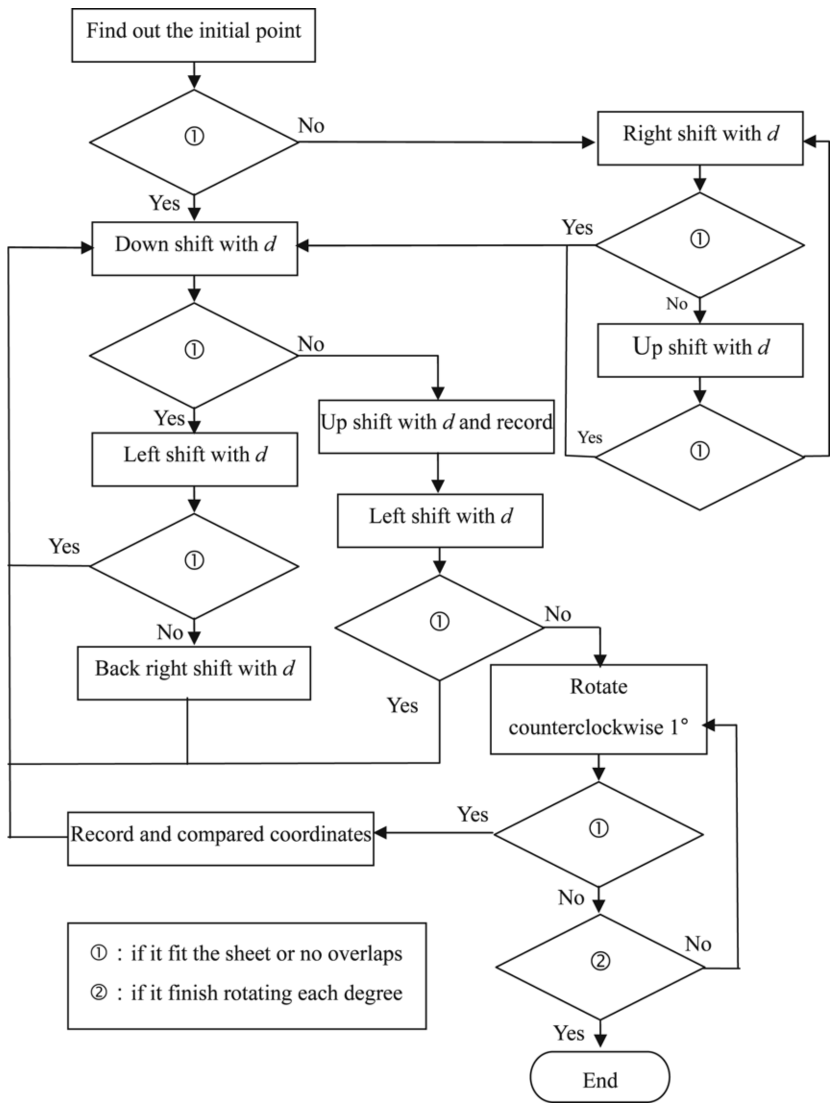

Fig. 7. The flow chart of the nest rules.

processed on a laptop computer with an Intel Pentium M processor operating at 1.73 GHz. The compared results are displayed in Figs. 8–10. Fig. 8 reveals that there is no discernable difference in utilization efficiency with the change of division angle. An approximated polygon is closer to an original shape when the value of θ is small. Unfortunately, this produces a large number of points and requires significant time to calculate. On the other hand, if the value of θ is large, it would make the approximated shape too large, affecting the nesting results. Therefore, it is better to use a division angle of 20°.

Utilization calculation is done by using Eq. (4). The rectangular zone is viewed as the used area of the sheet, and the width and length are decided by the outer coordinates of all parts.

$$\text{Utilizations} = \frac{\sum_{i=1}^{n} A_i}{R} \tag{4}$$

A_i: the area of the i part

R: the used area of the sheet.

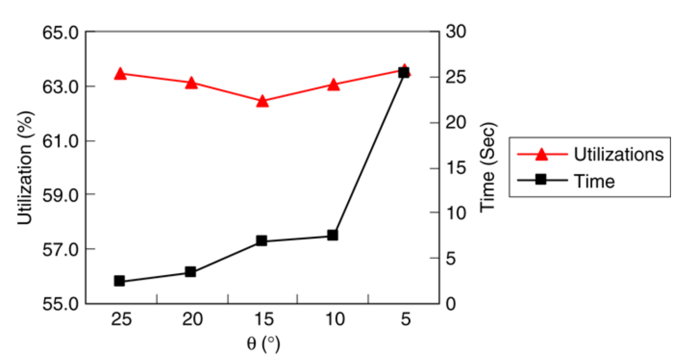

Fig. 8. The results of nesting with different θ.

As shown in Fig. 9, utilization decreases significantly if α is more than 0.25. Furthermore, the run time increases when α is too small. Therefore, the range of α is best suited to be between 0.02 and 0.20.

In addition, Fig. 10 shows that utilization decreases substantially when β is more than 1. There is no distinct

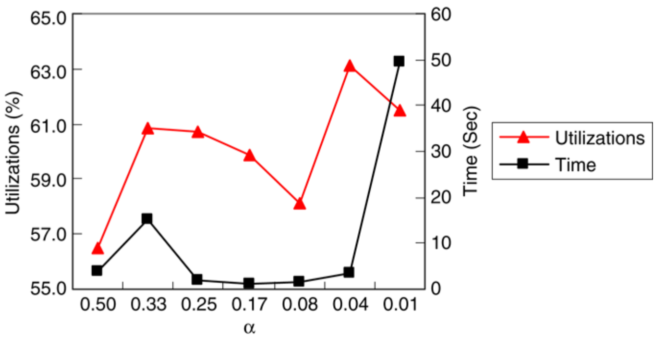

Fig. 9. The results of nesting with different values of α.

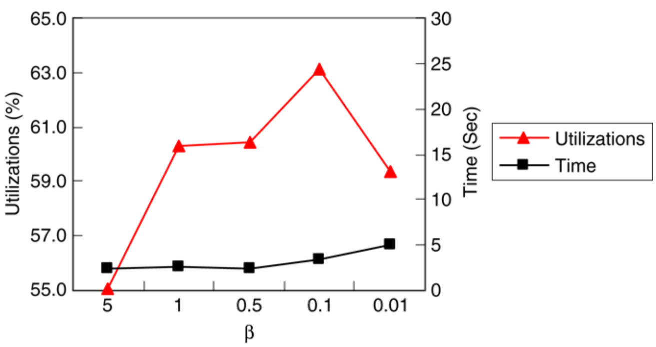

Fig. 10. The results of nesting with different values of β.

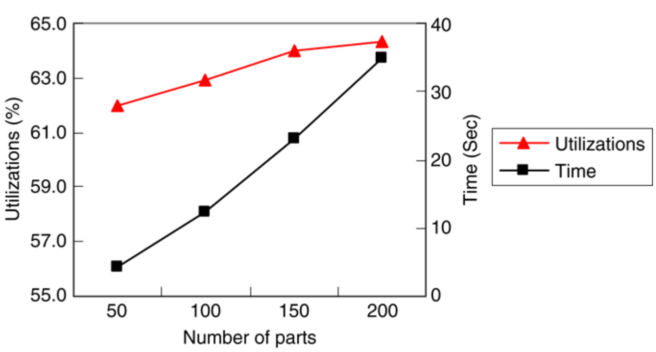

Fig. 11. The results of nesting with different numbers of parts.

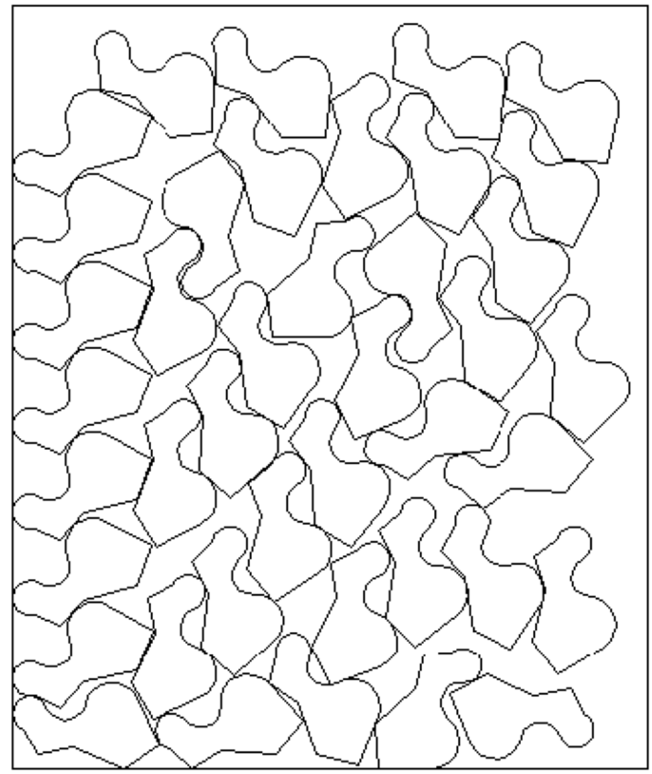

Fig. 12. The result of nesting with a single part on a fixed size of sheet.

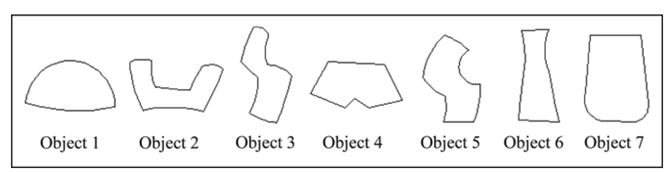

Fig. 13. Geometries of parts considered in the test problem.

variation in run time when changing β. Hence, β is best suited to be between 0.01 and 1.

Fig. 11 summarizes the results from other trials implemented to test the influences when nesting different numbers of parts. As can be seen, the results show that the run time is an arithmetic progression and the utilizations increase with the growth in the number of parts. This is because the approach this research proposes is nesting piece by piece.

Fig. 12 summarizes the results from trials conducted with a fixed sheet size. The utilization is 63.8%, the number of arranged parts is 41, θ is 20°, α is 0.04, and β is 0.1.

There existed no better way to measure nesting algorithms' performance, and, so this study evaluated some examples proposed by other researchers. A set of seven kinds of parts were tested by Wu et al. [20]. In the present study, the researchers scanned the parts and redrew them as shown in Fig. 13. Small errors could potentially have been introduced due to the scanning process. Four trials were conducted, and the trial conditions, parameters, and subsequent results are summarized in Table 1 and Figs. 14(a)–14(d). Wu et al. [20] fails to mention the run time of their trials. In addition, their utilization calculations were different from that of this study. In this case, there were seven kinds of parts, whose shapes were suited to the method of [20] because some part shapes were closed rectangles and the coefficient of variation of length was small. Figs. 14(a)–14(d) reveal that the arranged parts' concentration using the proposed approach is greater than the results obtained by [20]. Furthermore, the method presented here arranges these parts without any previous orientation or combination of parts, which contrasts with method [20].

The present study also tested a set of shapes proposed by Yousef [21], which were scanned and redrew as shown in Fig. 15. The results from four conducted trials are displayed in Table 2. The results from Table 3 show that both utilization and run time using QLM are better than those by [21]. There is a distinct gap in run time obtained between the two algorithms. The error from scanning can be ignored because the utilization calculations used the areas of the redrawn shapes. Fig. 16 depicts the plot of allocation achieved by QLM.

Table 1
The conditions and results of nesting (1)

Parts	Demand information for parts in test problems			
	Example 1	Example 2	Example 3	Example 4
Obj1	1	1	2	2
Obj2	3	4	1	2
Obj3	5	2	4	3
Obj4	2	2	3	1
Obj5	3	3	2	4
Obj6	5	2	2	5
Obj7	5	4	4	2
# of elements	24	18	18	19
Coefficient of variation of area	0.845	0.291	0.288	0.245
Coefficient of variation of length	0.04	0.041	0.04	0.037
Plate size	205 × 110	165 × 110	225 × 80	160 × 115
θ (°)	20	20	20	20
α	0.125	0.1	0.05	0.1
β	0.1	0.01	0.01	0.1
CPU time (s)	1.251	0.219	0.437	0.125

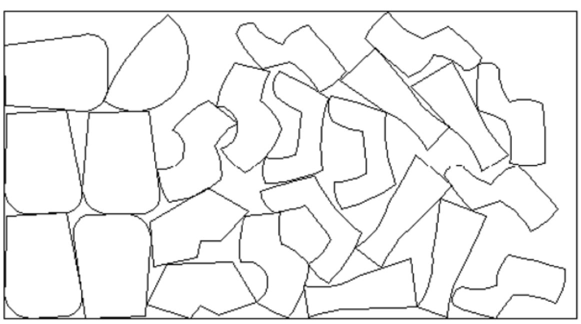

Fig. 14(a). Example 1 of compared results (1).

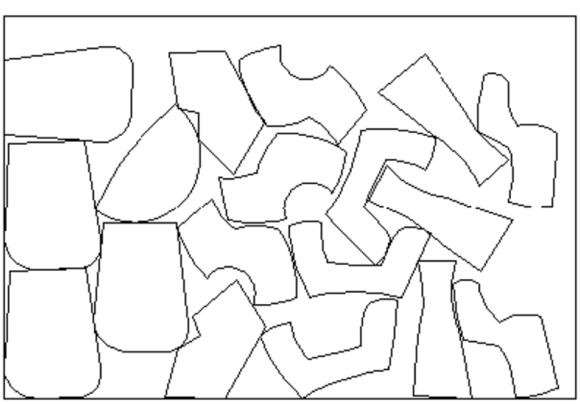

Fig. 14(b). Example 2 of compared results (1).

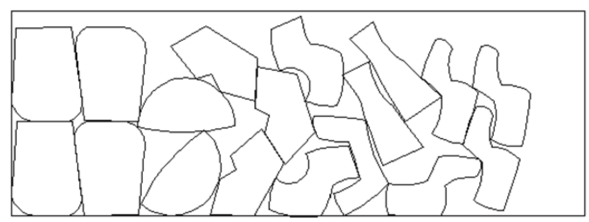

Fig. 14(c). Example 3 of compared results (1).

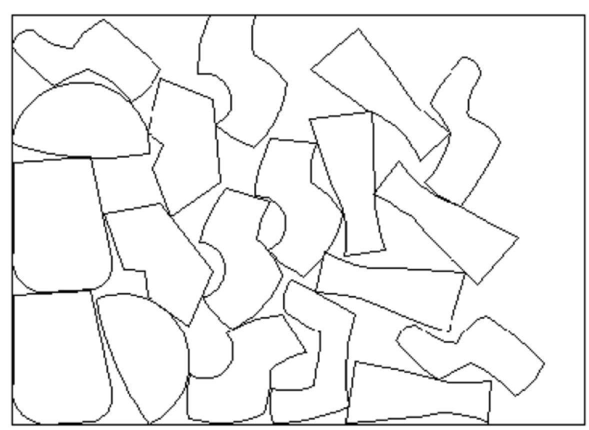

Fig. 14(d). Example 4 of compared results (1).

Babu and Babu [18] propose an example of arranging certain artistic shapes with irregular geometry on a sheet with an irregular boundary. The present study scanned, redrew and arranged these shapes as well. The tests were under different computer environments, and the results are summarized in Table 4. As shown, the shortest time is 5.578 s in trial 4. In

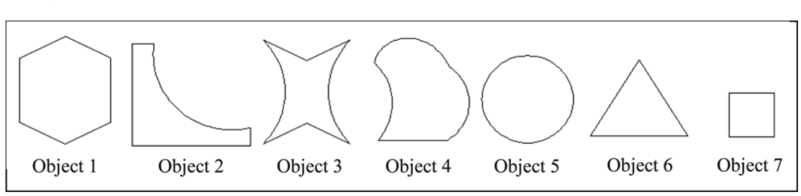

Fig. 15. Geometries of parts considered in the test problem.

Table 2
The conditions and results of nesting (2)

Parts	Demand information for parts in test problems			
	Trial 1	Trial 2	Trial 3	Trial 4
# of elements	28	28	28	28
Plate size	15 × 26.2	15 × 26.05	15 × 25.9	15× 26.1
θ (°)	20	20	20	20
α	0.15	0.15	0.075	0.025
β	0.05	0.01	0.01	0.01
CPU time (s)	0.422	0.391	1.125	8.64
Utilizations (%)	74.8	75.2	75.6	75.1

Table 3
The compared result (3) using different methods

Parts	Demand information for parts in test problems		
	QLM	Tabu search	HBH
Plate size	15× 25.9	15× 20.096	15 × 27.12
CPU time	1.125 s	2–10 min	2–10 min
Utilizations (%)	75.6	69.1	71.2

Table 4
The conditions and compared results (4)

Parts	Demand information for parts in test problems			
	Trial 1	Trial 2	Trial 3	Trial 4
# of elements	20	20	20	20
θ (°)	20	20	20	20
α	0.2	0.15	0.15	0.018
β	0.1	0.1	0.05	0.05
CPU time (s)	7.671	7.547	7.969	5.578
Utilizations (%)	51.3	51.5	51.6	50.8

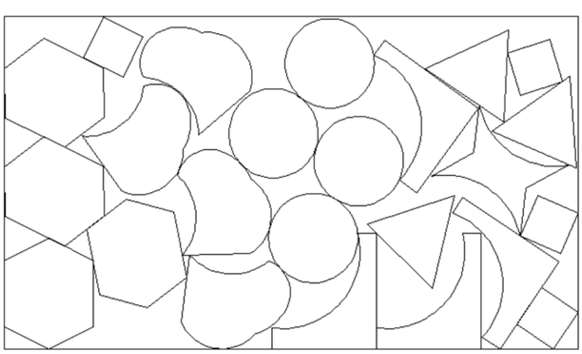

Fig. 16. The plot of compared results (2).

comparison, the same objects took 1230 s according to the data in [18]. QLM run time results are much shorter than those obtained by [18]. Unfortunately, the utilizations cannot be compared directly because the method of calculation is different. However, the utilization of QLM is better than that of [18] when adopting the same equation. Figs. 17(a) and 17(b) show the plots of two nesting results.

For the performance testing of the QLM algorithm, three examples proposed by [20,21,18] were tested and compared. QLM is comparable or superior to these algorithms both in utilization and process time from the results shown in Figs. 14(a)–14(d), 16, 17(a) and 17(b), and Tables 1–4.

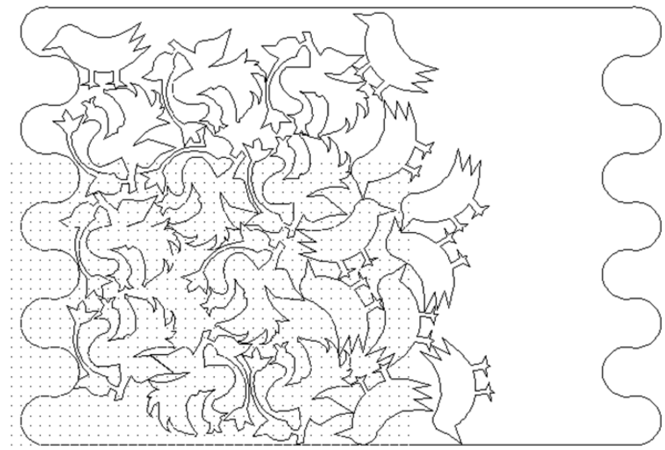

Fig. 17(a). Trial 2 of compared result (3).

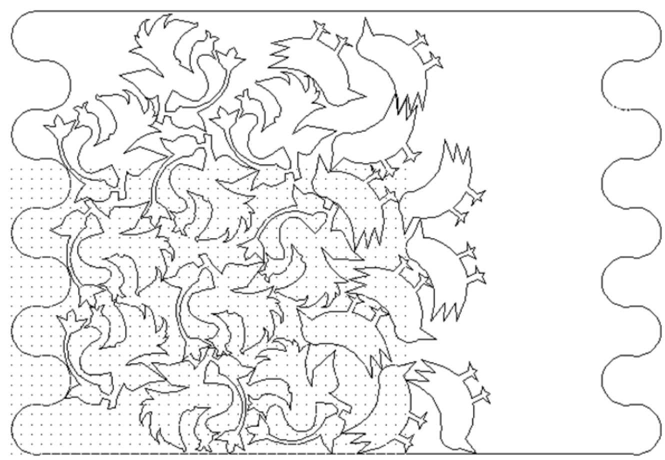

Fig. 17(b). Trial 3 of compared result (3).

With the exception of [18], the referred studies in Section 1 do not consider the situation where parts and sheets are multiple complexes with internal features or defective regions. QLM is capable of solving nesting problems for multiple two-dimensional complex irregular parts on multiple two-dimensional complex irregular sheets.

Finally, the nesting problem of multiple two-dimensional complex irregular parts on multiple two-dimensional complex irregular sheets were tested. There were four kinds of sheets and the results are shown in Fig. 18. Fig. 18 reveals that QLM works for this kind of complicated problem. Moreover, sheet 1 of Fig. 18 illustrates that QLM validly fills hole due to the design of division point.

4. Conclusions

This paper proposes the algorithm, quick location and movement (QLM), for solving irregular parts nesting on multiple irregular sheets. This algorithm includes two major parts: it approximates an irregular shape to a polygon with the use of a cluster of straight lines, and it arranges the approximated shapes one-by-one with the proposed step-by-step rule. This algorithm has been developed using a program using the C++ programming language. The program loads an

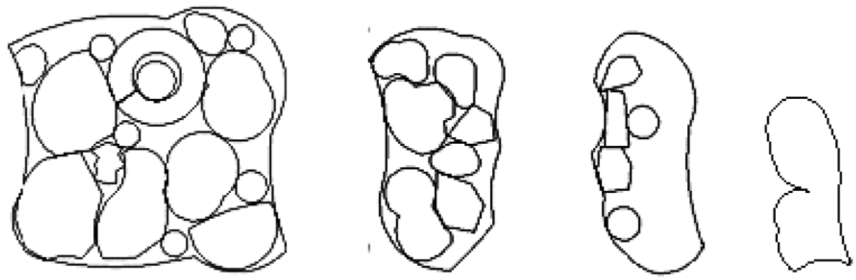

Fig. 18. The plot of multiple irregular parts on multiple irregular sheets.

AutoCAD file with the format of DXF (file_name.dxf), and the results are output to a file with the same format. This software technology is directly scalable to industrial application.

The arrangement efficiencies depend on the complexity, size and numbers of parts and sheets. Apart from solving nesting problems involving rectangular parts and sheets, the presented approach can also be extended directly to the arrangement of parts and sheets with highly irregular shapes. Inner waste filling and multiple sheets are also considered. The design of θ, α, and β makes it suitable for different areas and shapes of parts and sheets, and the result can be repeated and checked step-by-step because the results are the same whenever the parameters are fixed. The approach presented here is applicable to any kind of two-dimensional cutting stock problem. Although the present method has only been applied to shapes described by lines and arcs, the discrete representation used in this work can easily incorporate free-form features for the sheets and parts. In addition, the cutting tools and relevant technology or limitations therein are not considered when developing the approach.

The proposed approach has been compared with some methods proposed by other researches. The results reveal that QLM is better than other algorithms presented in the literature. The method we propose here is valid for any set of irregular parts and sheets, regardless of the number of pieces or piece types. Furthermore, it is more effective and efficient when applied for highly irregular parts and sheets.

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